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TWO-PHASE MULTICOMPONENT MASS TRANSPORT IN A DESCENDING,

STRAIGHT-THROUGH FLOW OF PHASES

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The parameters of combined, multicomponent mass transport in a two-phase gas (vapor)—liquid system moving as a descending, laminar straight-through flow are investigated theoretically, and methods for calculating them are proposed. The methods are based on solving the system of differential equations for multicomponent convective diffusion in both phases with an allowance for the conjugation conditions at interface. The diffusion equations are solved both numerically and analytically in the boundary-layer approximation. The development of the process over large lengths of the contact device is investigated, and the asymptotic values of the component concentrations are determined.

Modern theory and industrial practice require adequate methods for investigating and calculating complex mass-exchange processes. The development of such methods, in turn, requires the tool of differential equations of convective mass transport [1]. When used for the description of multicomponent systems, these equations are characterized by matrix vectors and a conjugate form, which makes their solution difficult. For two-phase, multicomponent systems, the solution becomes considerably more complex, since the matrix conjugation equations which relate the component concentrations of both phases at the interface also have a conjugate character.

The present article presents a theoretical investigation of the mass-exchange process in a multicomponent, gas—liquid mixture, based on solving the system of differential equations of multicomponent convective diffusion in conjugate form.

Consider an n-component mixture moving in a rectangular channel as a descending, straight-through flow of phases. The x axis is oriented along the channel axis, while the y axis is perpendicular to it (Fig. 1). We make the usual assumptions [2-4]: The physical characteristics of the phases are constant, external forces are absent, phase equilibrium conditions prevail at the interface, and the thickness of the liquid film is constant. Then, the equations of convective, multicomponent diffusion are given by

$$u_\ell(y) \frac{\partial \mathbf{C}_\ell}{\partial x} = [D_\ell] \frac{\partial^2 \mathbf{C}_\ell}{\partial y^2}. \tag{1}$$

$$u_g(y) \frac{\partial \mathbf{C}_g}{\partial x} = [D_g] \frac{\partial^2 \mathbf{C}_g}{\partial y^2}, \tag{2}$$

while the boundary conditions are assigned by the following relationships:

at the channel inlet,

$$x = 0, \quad \mathbf{C}_\ell = \mathbf{C}_{0\ell}, \quad \mathbf{C}_g = \mathbf{C}_{0g}; \tag{3}$$

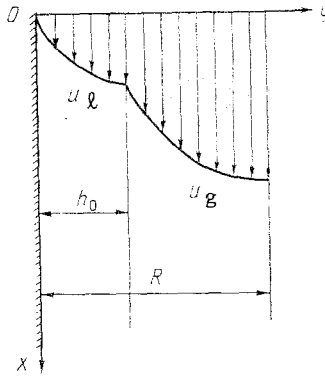at the channel wall (impenetrability condition),

Fig. 1. Coordinate system and velocity distributions in the phases.

$$y = 0, \quad \frac{\partial \mathbf{C}_\ell}{\partial y} = 0; \tag{4}$$

At the channel axis (symmetry conditions),

$$y = R, \quad \frac{\partial \mathbf{C}_g}{\partial y} = 0; \tag{5}$$

at the interface (conditions of phase equilibrium and continuity of flow of the components),

$$y = h_0, \quad \mathbf{C}_g = [M]\,\mathbf{C}_\ell; \tag{6}$$

$$\mathbf{J}_g = \mathbf{J}_\ell \tag{7}$$

Using the generalized Fick law, we reduce the latter equation to the following form:

$$[D_g]\,\frac{\partial \mathbf{C}_g}{\partial y} = [D_\ell]\,\frac{\partial \mathbf{C}_\ell}{\partial y}. \tag{8}$$

All matrices and vectors in Eqs. (1)-(8) are of the (n-1)-th order.

System (1)-(2) with the boundary conditions (3)-(6) and (8) constitutes a system of parabolic-type differential equations with boundary conditions of the fourth kind. The system cannot be solved in this form.

In order to transform the system, we shall utilize an important property of the matrices of diffusion coefficients $[D_\ell]$ and $[D_g]$, which consists in the possibility of reducing them to the diagonal form [2, 5]:

$$[L]^{-1}[D_\ell][L] = \ulcorner D'_\ell \lrcorner, \quad [G]^{-1}[D_g][G] = \ulcorner D'_g \lrcorner, \tag{9}$$

where $D'_{\ell i}$ and $D'_{g i}$ are the eigenvalues of matrices $[D_\ell]$ and $[D_g]$, which constitute real positive numbers.

By multiplying matrix $[L]^{-1}$ on the left by Eq. (1), and matrix $[G]^{-1}$ by Eq. (2), we obtain

$$u_\ell(y)\frac{\partial \mathbf{C}'_\ell}{\partial x} = \ulcorner D'_\ell \lrcorner \frac{\partial^2 \mathbf{C}'_\ell}{\partial y^2}; \quad u_g(y)\frac{\partial \mathbf{C}'_g}{\partial x} = \ulcorner D'_g \lrcorner \frac{\partial^2 \mathbf{C}'_g}{\partial y^2}, \tag{10}$$

where

$$\mathbf{C}'_\ell = [L]^{-1}\mathbf{C}_\ell, \quad \mathbf{C}'_g = [G]^{-1}\mathbf{C}_g. \tag{11}$$
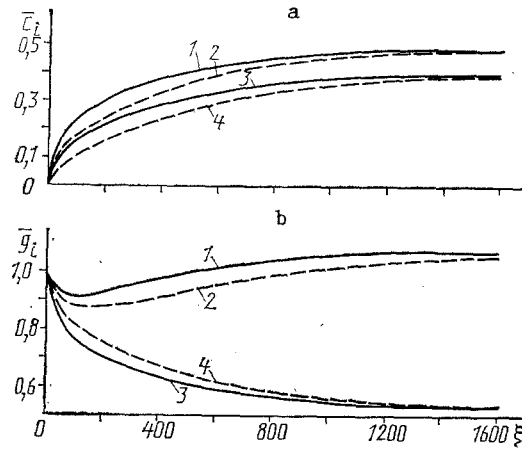
At the same time,

Fig. 2. Integral-mean dimensionless concentrations of components in the liquid (a) and the gaseous (b) phases as functions of the length of the contact device (the solid curves pertain to a flat velocity profile, while the dashed curves pertain to a parabolic profile). 1) and 2) First component; 3) and 4) second component.

$$\text{for } x = 0 \quad C'_\ell = C'_{0\ell} = [L]^{-1}C_{e\,\ell}, \quad C'_g = C'_{0g} = [G]^{-1}C_{0g}; \tag{12}$$

$$\text{for } y = 0 \quad \frac{\partial C'_\ell}{\partial y} = 0; \quad \text{for } y = R \quad \frac{\partial C'_g}{\partial y} = 0; \tag{13}$$

$$\text{for } y = h_0 \quad C'_g = [R]\,C'_\ell, \text{ where } [R] = [G]^{-1}[M][L]; \tag{14}$$

$$[D_g]\,[G]\,\frac{\partial C'_g}{\partial y} = [D_\ell]\,[L]\,\frac{\partial C'_\ell}{\partial y}. \tag{15}$$

In the derived system (10)-(15), the connectedness of the equations persists only in the boundary conditions at the interface phases between (14) and (15), while the equations of convective mass transport (10) and (11) can be considered as systems of independent differential equations of the $(n - 1)$-th order.

We introduce dimensionless variables by means of the relationships

$$x = \frac{\mathrm{Re}_\ell h_0}{4}\xi; \quad y = h_0\eta_1\,(0 \leqslant y \leqslant h_0); \quad y = R - (R - h_0)\,\eta_2\,(h_0 \leqslant y \leqslant R);$$

$$u_\ell = \bar{u}_\ell v_\ell(\eta_1); \quad u_g = \bar{u}_g v_g(\eta_2); \tag{16}$$

$$C'_\ell = C'_{0\ell} + \ulcorner\Delta_\ell\lrcorner c; \quad C'_g = C_{eg} - \ulcorner\Delta_g\lrcorner g,$$

where

$$\Delta_{\ell k} = C'_{e\ell k} - C'_{0\ell k}; \quad \Delta_{gk} = C'_{egk} - C'_{0gk}, \quad k = 1,\ 2,\ \ldots,\ n-1;$$

$$C'_{e\ell} = [R]^{-1}C'_{0g}; \quad C'_{eg} = [R]\,C'_{0\ell}.$$

Then, system (10)-(15) assumes the final form

$$v_\ell(\eta_1)\,\frac{\partial c}{\partial \xi} = \ulcorner Sc_\ell\lrcorner^{-1}\frac{\partial^2 c}{\partial \eta_1^2}; \quad \beta v_g(\eta_2)\,\frac{\partial g}{\partial \xi} = \ulcorner Sc_g\lrcorner^{-1}\frac{\partial^2 g}{\partial \eta_2^2}, \tag{17}$$
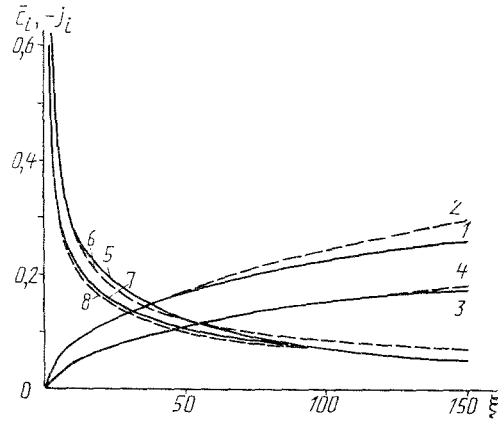
Fig. 3. Integral-mean dimensionless component concentrations in the liquid phase (1-4) and dimensionless mass fluxes of components (5-8) as functions of the length of the contact device (solid curves pertain to the numerical solution, while dashed curves refer to the analytical solution). 1), 2), 5), and 6) First component; 3), 4), 7), and 8) second component.

where

$$\mathrm{Sc}'_{\varrho k} = v_\varrho / D'_{\varrho k}; \quad \mathrm{Sc}'_{\mathrm{g}k} = v_\mathrm{g} / D'_{\mathrm{g}k}, \quad k = 1, 2, \ldots, n-1; \quad \beta = \frac{\mathrm{Re}_\mathrm{g}(R - h_0)}{\mathrm{Re}_\varrho h_0}. \tag{18}$$

For $\xi = 0$ $c = 0$, $g = e$ (e is a vector consisting of unit elements); $\tag{19}$

$$\text{for } \eta_1 = 0 \quad \partial c / \partial \eta_1 = 0; \text{ for } \eta_2 = 0 \quad \partial g / \partial \eta_2 = 0; \tag{20}$$

$$\text{for } \eta_1 = \eta_2 = 1 \quad g = [Q] \, c, \text{ where } [Q] = -\ulcorner \Delta_\mathrm{g} \lrcorner^{-1} [R] \ulcorner \Delta_\varrho \lrcorner; \tag{21}$$

$$\frac{\partial g}{\partial \eta_2} = [M_\varepsilon] \frac{\partial c}{\partial \eta_1}, \text{ where } [M_\varepsilon] = \frac{R - h_0}{h_0} \ulcorner \Delta_\mathrm{g} \lrcorner^{-1} [G]^{-1} [D_\mathrm{g}]^{-1} [D_\varrho] [L] \ulcorner \Delta_\varrho \lrcorner. \tag{22}$$

It should be noted that matrix $[Q]$ has a remarkable property: The sum of elements of each of its rows is equal to unity. Actually,

$$\sum_{j=1}^{n-1} [Q]_{hj} = -\frac{1}{\Delta_{\mathrm{g}_k}} \sum_{j=1}^{n-1} [R]_{hj} \Delta \cdot \varrho_j = -\frac{1}{\Delta_{\mathrm{g}_k}} \left\{ \sum_{j=1}^{n-1} [R]_{hj} C'_{e \varrho_j} - \sum_{j=1}^{n-1} [R]_{hj} C'_{0 \varrho_j} \right\} =$$

$$= -\frac{1}{\Delta_{\mathrm{g}_k}} (C'_{0\mathrm{g}_k} - C'_{e\mathrm{g}_k}) = 1.$$

The relationship between the flow velocities of the phases and the transverse coordinate figuring in the convective diffusion equations can assume different forms. We shall subsequently consider the flow of phases at mean-dsicharge velocities (flat profiles), where

$$v_\varrho(\eta_1) = \bar{v}_\varrho = 1; \quad v_\mathrm{g}(\eta_2) = \bar{v}_\mathrm{g} = 1, \tag{23}$$

and also flow at velocities obeying the parabolic law (Fig. 1):

$$v_\varrho(\eta_1) = s_1 + a_1 \eta_1 + b_1 \eta_1^2; \quad v_\mathrm{g}(\eta_2) = s_2 + a_2 \eta_2 + b_2 \eta_2^2. \tag{24}$$

The coefficients of relationships (24) can be found from the hydrodynamic conditions
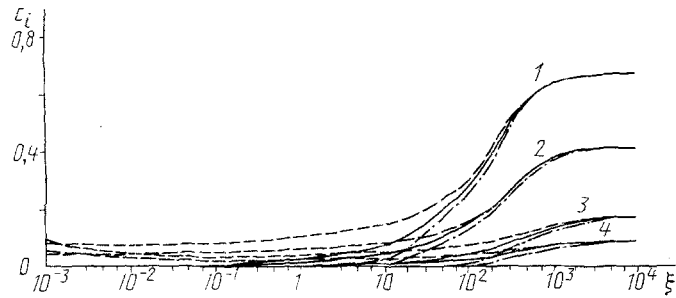
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Fig. 4. Dimensionless component concentrations in the liquid phase as functions of the length of the contact device for different values of $Sc'_{\ell i}$ (the solid curves pertain to values in the core of phase, the dashed curves refer to those at the interface, and the dash-dot curves pertain to values at the wall). 1) and 2) $Sc'_{\ell 1} = 70.0$; $Sc'_{\ell 2} = 90.0$ (1, c_1; 2, c_2); 3) and 4) $Sc'_{\ell 1} = 700.0$; $Sc'_{\ell 2} = 900.0$ (3, c_1; 4, c_2).

$$y = 0, \quad u_\ell = 0; \quad y = R, \quad \partial u_g/\partial y = 0;$$

$$y = h_0, \quad u_\ell = u_g, \quad \mu_\ell \frac{\partial u_\ell}{\partial y} = \mu_g \frac{\partial u_g}{\partial y},$$

and also from the definitions of the mean velocities of the liquid and the gas:

$$\frac{1}{h_0} \int_0^{h_0} u_\ell dy = \bar{u}_\ell; \quad \frac{1}{R - h_0} \int_{h_0}^{R} u_g dy = \bar{u}_g,$$

which, in dimensionless form, are given by

$$\eta_1 = 0, \quad v_\ell = 0; \quad \eta_2 = 0, \quad \frac{\partial v_g}{\partial \eta_2} = 0;$$

$$\eta_1 = \eta_2 = 1, \quad \varphi v_\ell = v_g, \quad \varkappa \frac{\partial v_\ell}{\partial \eta_1} = \frac{\partial v_g}{\partial \eta_2};$$

$$\bar{v}_\ell = \int_0^1 v_\ell d\eta_1 = 1; \quad \bar{v}_g = \int_0^1 v_g d\eta_2 = 1,$$

(25)

where

$$\varphi = \frac{\bar{u}_\ell}{\bar{u}_g}; \quad \varkappa = - \frac{\mu_\ell \bar{u}_\ell (R - h_0)}{\mu_g \bar{u}_g h_0}.$$

With an allowance for (25), relationships (24) assume the following form:

$$v_\ell(\eta_1) = \frac{3}{4\varkappa - 3\varphi} [2(2\varkappa - 3\varphi + 1)\eta_1 + (6\varphi - 2\varkappa - 3)\eta_1^2];$$

(26)

(27)

$$v_g(\eta_2) = \frac{3}{4\varkappa - 3\varphi} [(2\varkappa - \varphi - \varkappa\varphi) + \varkappa(3\varphi - 2)\eta_2^2].$$

Thus, the solution of our dimensionless problem (16)-(22), (26), and (27) is determined by the dimensionless parameters φ, $\varkappa$, β, $\ulcorner Sc'_{\ell} \lrcorner$, Sc'_{g}, $[Q]$ and $[M_E]$, i.e., it depends on $2n(n-1) + 3$

quantities. In the case of a flat profile, the number of determining is equal to $2n(n-1) + 1$.

The solution is obtained by means of the trial-and-error method, using finite-difference operators (three-point pattern for the implicit scheme [6]):

$$\frac{\partial W}{\partial \xi} = \frac{W(i,\ j) - W(i-1,\ j)}{\Delta \xi}, \quad \frac{\partial W}{\partial \eta} = \frac{W(i,\ j) - W(i,\ j-1)}{\Delta \eta},$$

$$\frac{\partial^2 W}{\partial \eta^2} = \frac{W(i,\ j-1) - 2W(i,\ j) + W(i,\ j+1)}{(\Delta \eta)^2},$$

where i and j are the numbers of steps along the vertical and the horizontal, respectively, $\Delta \xi$ and $\Delta \eta$ are the pertinent magnitudes of the steps, and W is the concentration of the k-th component in the liquid or the gaseous phase.

Since the relationships between the dimensionless component concentrations and their derivatives at the interface between phases have a conjugate character (expressions (21) and (22)), determination of the boundary trial coefficients requires the solution of a system of linear algebraic equations at each step along the vertical. In this, for greater accuracy, the derivatives at the interface are replaced by finite-difference analogs, which makes it possible to approximate them with an accuracy to small quantities of the third order [7]:

$$\frac{\partial W}{\partial \eta}\bigg|_{\eta=1} = \frac{1}{12(\Delta \eta)}\ [8f(i) - 18W(i,\ N-1) + W(i,\ N-2) + 9W(i,\ N)],$$

where N is the number of steps along the horizontal, while f(i) is the value of the corresponding function at the interface,

$$f(i) = \frac{W(i,\ N-1) + W(i,\ N)}{2}.$$

In order to calculate the parameters of multicomponent mass transport, it is necessary to determine the fundamental matrices [L] and [G], by means of which the relationships of diagonalization (9) are realized. As is known [8], the fundamental matrix of the [D] matrix consists of its right-hand column eigenvectors, defined for each eigenvalue D'_k by the relationship

$$[D]\, \mathbf{v}_k = D'_k \mathbf{v}_k. \tag{28}$$

If we multiply (28) by an arbitrary real nonzero constant ψ_k, we obtain

$$[D]\, \mathbf{z}_k = D'_k \mathbf{z}_k,$$

where $\mathbf{z}_k = \psi_k \mathbf{v}_k$ also is the right-hand column eigenvector [D] corresponding to D'_k.

Utilizing this property, we can find the nondegenerate solution of system (28). For this, we impose an additional condition on the set of right-hand column eigenvectors $\mathbf{v}_k$

$$(\mathbf{v}_k)_i = \delta_{ki}; \quad k,\ i = 1,\ 2,\ ...,\ n-1.$$

In other words, the arbitrary constants ψ_k are chosen so that every k-th element of the k-th eigenvector is equal to unity. Then, instead of the system of equations (28), we obtain the equivalent system

$$[D_k]\, \mathbf{z}_k = \mathbf{d}_k, \tag{29}$$

where

$$[D_k]_{ij} = ([D]_{ij} - D'_k \delta_{ij})(1 - \delta_{ki}) + \delta_{ki}\delta_{kj},$$

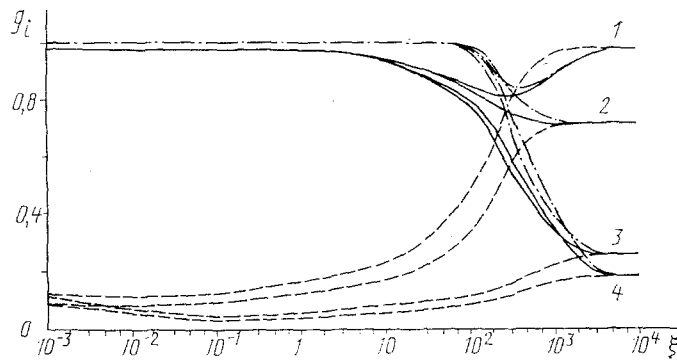$$(\mathbf{d}_k)_i = \delta_{ki}, \quad k,\ i,\ j = 1,\ 2,\ ...,\ n-1.$$

Fig. 5. Dimensionless component concentrations in the gaseous phase as functions of the length of the contact device (the parameter values and the notation are the same as in Fig. 4; the dash-dot curve pertains to values at the axis of the contact device). 1) and 3) g_1; 2) and 4) g_2.

For $i \neq k$, (29) becomes (28), while, for $i = k$, we obtain the simple equations

$$[D_h]_{kj} = \delta_{kj}, \quad (d_h)_k = 1,$$

and, consequently, $(z_h)_k = 1$.

System (29) is solved by means of the usual methods for each value of k, $k = 1, 2, \ldots,$ $n - 1$, which actually facilitates the writing of the fundamental matrix of the [D] matrix.

The proposed method of determining the fundamental matrix is applicable to matrices without multiple eigenvalues, which is virtually always the case for multicomponent diffusion processes. It can be proven that the system of dimensionless parameters determining the solution of the problem is invariant with regard to the choice of the fundamental matrices [L] and [G].

The distributions of dimensionless concentrations in the phases $c(\xi, \eta_1)$ and $g(\xi, \eta_2)$ are the result of the numerical solution of the system of differential equations for convective diffusion (17). They can be used for determining any characteristic of two-phase, multicomponent mass transport, for instance, the component concentrations at the wall and the axis of the channel and at the interface; the integral-mean values, defined by the expressions

$$\overline{c}(\xi) = \int_0^1 v_\ell(\eta_1) c(\xi, \eta_1) d\eta_1; \quad \overline{g}(\xi) = \int_0^1 v_g(\eta_2) g(\xi, \eta_2) d\eta_2, \tag{30}$$

interphase component fluxes, etc.

As an example, we shall consider three-component mass exchange in a two-phase system characterized by the following dimensionless parameters $\varphi = 0.01$; $\varkappa = -12.73$; $\beta = 1410.35$; $Sc'_{\ell 1} = 70.82$; $Sc'_{\ell 2} = 91.39$; $Sc'_{g1} = 0.81$; $Sc'_{g2} = 1.04$;

$$[Q] = \begin{bmatrix} 8.24 & -7.24 \\ 1.30 & -0.30 \end{bmatrix}; \quad [M_\varepsilon] = \begin{bmatrix} -43.90 & 44.02 \\ -19.76 & -0.13 \end{bmatrix}.$$

In principle, such parameters can describe any gas or vapor–liquid mass-exchange process in a three-component mixture, in particular, three-component film distillation or isothermic absorption by the nonvolatile film of the absorbent of a two-component gaseous mixture in the presence of an inert component (see [5]). Figure 2 shows the relationships between the integral-mean dimensionless concentrations of components and the length of the contact device in the liquid and the gaseous phases.
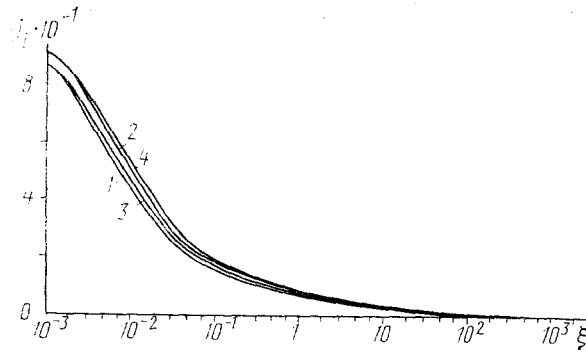
Fig. 6. Dimensionless component mass fluxes as functions of the length of the contact device (the parameter values are the same as in Fig. 4). 1) and 3) j_1; 2) and 4) j_2.

In view of the fact that the mass transport process under consideration has a boundary-layer character, the gradients of component concentrations near the interface reach considerable values. In particular, this pertains to the mass-exchange segments located at small distances ξ from the inlet of the contact device. In view of this, we can substitute the following two conditions for the boundary conditions at the wall and the axis of the contact device (see [4]):

$$c_\ell = c_{0\cdot\ell} \quad \text{for} \quad y \to -\infty; \quad c_g = c_{0g} \quad \text{for} \quad y \to \infty \tag{31}$$

and obtain the analytical solution of the problem in the boundary-layer approximation.

Let $\eta_3 = 1 - \eta_1$ and $\eta_4 = 1 - \eta_2$. The velocities of the liquid and the gaseous phases are assumed to be equal to their integral-mean values (see (23)):

$$v_\ell = \overline{v}_\ell = 1; \quad v_g = \overline{v}_g = 1.$$

With an allowance for these relationships, the problem in dimensionless form is defined as follows:

$$\frac{\partial c}{\partial \xi} = \ulcorner Sc'_{\ell \lrcorner}{}^{-1} \frac{\partial^2 c}{\partial \eta_3^2}; \quad \beta \frac{\partial g}{\partial \xi} = \ulcorner Sc'_{g \lrcorner}{}^{-1} \frac{\partial^2 g}{\partial \eta_4^2}; \tag{32}$$

$$\text{for} \quad \xi = 0 \quad c = 0, \ g = e; \tag{33}$$

$$\text{for} \quad \eta_3 = \eta_4 = 0 \quad g = [Q] c, \quad \frac{\partial g}{\partial \eta_4} = [M_\varepsilon] \frac{\partial c}{\partial \eta_3}; \tag{34}$$

$$\text{for} \quad \eta_3 \to \infty \quad c = 0; \quad \text{for} \quad \eta_4 \to \infty \quad g = e. \tag{35}$$

System (33)-(35) is solved by using the analytical method proposed in [1, 9], while the resulting distributions of the dimensionless component concentrations are given by

$$c = (\ulcorner I_\lrcorner - \ulcorner S'_{\ell \lrcorner}) [\alpha] e; \quad g = (\ulcorner I_\lrcorner - \ulcorner S'_{g \lrcorner}) [Q] [\alpha] e + \ulcorner S'_{g \lrcorner} e, \tag{36}$$

$$S'_{\ell i}(\xi, \eta_3) = \text{erf}(z_{1i}), \quad z_{1i} = \frac{\eta_3}{2} \sqrt{\frac{Sc'_{\ell i}}{\xi}}; \tag{37}$$

$$S'_{g i}(\xi, \eta_4) = \text{erf}(z_{2i}), \quad z_{2i} = \frac{\eta_4}{2} \sqrt{\frac{Sc'_{g i}\beta}{\xi}}; \tag{38}$$

$$[\alpha] = ([Q] - [\Psi])^{-1}, \quad [\Psi] = \frac{1}{\sqrt{\beta}} \ulcorner Sc'_{g \lrcorner}{}^{-0,5} [M_\varepsilon] \ulcorner Sc'_{\ell \lrcorner}{}^{0,5}. \tag{39}$$

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If we obtain successively the inverse transforms of relationships (36)-(39) by means of the expressions (16) for introducing dimensionless variables and the diagonalization expressions (11), we arrive at expressions for the mole concentrations C_ℓ and C_g that are equivalent to those derived in [1, 9].

The distributions of the component concentrations (36) make it possible to determine the dimensionless fluxes of components through the interphase surface. For this, we differentiate expressions (36) at the points $\eta_3 = 0$, and $\eta_4 = 0$, respectively:

$$\frac{\partial c}{\partial \eta_3}\bigg|_{\eta_3=0} = -\frac{1}{\sqrt{\pi\xi}}\, ^\ulcorner Sc_{\ell\lrcorner}'{}^{0,5}\, [\alpha]\, e;$$

$$\frac{\partial g}{\partial \eta_4}\bigg|_{\eta_4=0} = -\sqrt{\frac{\beta}{\pi\xi}}\, ^\ulcorner Sc_{g\lrcorner}'{}^{0,5}\,([Q]\,[\alpha] - \,^\ulcorner I_\lrcorner)\,e. \tag{40}$$

In correspondence with (34), we define the dimensionless flux $\mathbf{j}$ by the expression

$$\mathbf{j} = \frac{\partial g}{\partial \eta_4}\bigg|_{\eta_4=0} = [M_\varepsilon]\,\frac{\partial c}{\partial \eta_3}\bigg|_{\eta_3=0}. \tag{41}$$

In passing to dimensional flux values, we find from (40) and (41)

$$\mathbf{J}_\ell = \mathbf{J}_g = \left(\sqrt{\frac{\overline{u_g}}{\pi x}}\,[D_g]^{-0,5} + \sqrt{\frac{\overline{u_\ell}}{\pi x}}\,[M]\,[D_\ell]^{-0,5}\right)^{-1}([M]\,\mathbf{C}_{c\,\ell} - \mathbf{C}_{0g}).$$

The latter expression coincides with that derived in [1, 9].

Figure 3 shows the distributions of the dimensionless integral-mean component concentrations in the liquid phase and of the dimensionless mass fluxes of components, which have been obtained by means of the numerical method (flat velocity profile) and also expressions (36)-(39) and (41). The values of the problem parameters are the same as before (see Fig. 2).

Analysis shows that the analytical solution (36) is relatively adequate for describing the process of two-phase, multicomponent mass exchange over the initial segment at the contact device, where there is no appreciable effect of mass transport on the component concentrations at the wall (liquid phase) or at the channel axis (gaseous phase). Therefore, if the length of the contact device does not exceed a certain given value ξ_a ($\xi_a \cong \min\limits_{h}\{Sc'_{\ell h}\}$), we can use expressions (36)-(39). If $\xi > \xi_a$, the numerical method described above should be used.

We shall now consider the development of the process over large lengths. If $\xi \to \infty$, the concentration gradients gradually diminish, the process of mass transport slows down (see Fig. 2), and the component concentrations at the interface and in the cores of phases tend to the same asymptotic values.

In order to determine the asymptotic behavior of the component concentrations for $\xi \to \infty$, we use the relationships

$$\frac{d}{d\xi}\int_0^1 v_\ell c_h\,(\xi,\ \eta_1)\,d\eta_1 = -\frac{1}{Sc'_{\ell h}}\,\frac{\partial c_h}{\partial \eta_1}\bigg|_{\eta_1=1};$$

$$\beta\,\frac{d}{d\xi}\int_0^1 v_g g_h\,(\xi,\ \eta_2)\,d\eta_2 = \frac{1}{Sc'_{gh}}\,\frac{\partial g_h}{\partial \eta_2}\bigg|_{\eta_2=1},\quad k=1,\ 2,\ \ldots,\ n-1, \tag{42}$$

which can be obtained by integrating the equations of convective diffusion in the phases along the thickness of the film and the gas layer, respectively, with an allowance for the phase continuity equations.

We have the following from boundary condition (22):

$$\mathrm{Sc}_{g_2}^{\prime -1} \frac{\partial \mathbf{g}}{\partial \eta_2}\bigg|_{\eta_2=1} = \ulcorner \mathrm{Sc}_{g_{\lrcorner}}^{\prime -1} [M_\varepsilon] \frac{\partial \mathbf{c}}{\partial \eta_1}\bigg|_{\eta_1=1} = [\Omega] \ulcorner \mathrm{Sc}_{\ell_{\lrcorner}}^{\prime -1} \frac{\partial \mathbf{c}}{\partial \eta_1}\bigg|_{\eta_1=1}, \tag{43}$$

where

$$[\Omega] = \ulcorner \mathrm{Sc}_{g_{\lrcorner}}^{\prime -1} [M_\varepsilon] \ulcorner \mathrm{Sc}_{\ell_{\lrcorner}}^{\prime}. \tag{44}$$

With an allowance for (43), we obtain from (42)

$$\beta \frac{d}{d\xi} \int_0^1 v_g \mathbf{g}(\xi, \ \eta_2) \, d\eta_2 = [\Omega] \frac{d}{d\xi} \int_0^1 v_\ell \mathbf{c}(\xi, \ \eta_1) \, d\eta_1$$

or (see (30))

$$\frac{d}{d\xi} (\beta \bar{\mathbf{g}} - [\Omega] \bar{\mathbf{c}}) = 0. \tag{45}$$

It follows from (45) that the expression within brackets is constant for any ξ. In particular if $\xi = 0$, when conditions (19) are satisfied, this expression is equal to βe and, consequently

$$\beta \bar{\mathbf{g}} - [\Omega] \bar{\mathbf{c}} = \beta \, \mathbf{e}. \tag{46}$$

Equation (46) represents the dimensionless form of the mass balance for a contact device segment of arbitrary length, starting at $\xi = 0$.

We now consider that, for $\xi \to \infty$,

$$\bar{\mathbf{c}} = \mathbf{c}|_{\eta_1=1}; \quad \bar{\mathbf{g}} = \mathbf{g}|_{\eta_2=1},$$

and use boundary conditions (21):

$$\bar{\mathbf{g}} = [Q] \bar{\mathbf{c}}. \tag{47}$$

Then, as a result of simultaneous solution of (46) and (47), we obtain the sought asymptotic values of the vectors of component concentrations:

$$\bar{\mathbf{c}}(\xi \to \infty) = \left([Q] - \frac{1}{\beta} [\Omega]\right)^{-1} \mathbf{e}; \tag{48}$$

$$\bar{\mathbf{g}}(\xi \to \infty) = [Q] \left([Q] - \frac{1}{\beta} [\Omega]\right)^{-1} \mathbf{e}. \tag{49}$$

As an example, we shall consider the two-phase absorption of a three-component mixture where one of the components is inert. Figures 4 and 5 show the characteristic relationship between the dimensionless component concentrations in the liquid and the gaseous phases and the length of the contact device, plotted for a parabolic velocity profile and the following values of the dimensionless parameters: $\varphi = 0.01$; $\varkappa = -12.0$; $\beta = 1400.0$; $\mathrm{Sc}'_{g_1} = 0.8$; $\mathrm{Sc}'_{g_2} = 1.0$;

$$[Q] = \begin{bmatrix} 2.25 & -1.25 \\ 1.30 & -0.30 \end{bmatrix}; \quad [M_\varepsilon] = \begin{bmatrix} -40.0 & 50.0 \\ -20.0 & 15.0 \end{bmatrix}.$$

The solid curves pertain to concentration profiles in the phase core (integral-mean values), the dashed curves pertain to the profiles of surface values, while the dash-dot curves refer to profiles of values at the wall (liquid phase) and at the channel axis (gaseous phase).

The diagrams show clearly that the process reaches the limiting equilibrium values $\bar{c}(\xi=\infty)$ and $\bar{g}(\xi=\infty)$ (see (48) and (49)).

Figure 6 shows the variation corresponding to Figs. 4 and 5 of the dimensionless component mass fluxes in relation to the length of the device. The curves were plotted as a result of differentiation of the concentration profiles at the interface between phases.

In conclusion, it should be noted that the calculation methods proposed in the present article make it possible not only to determine the fields of component concentrations and the flux of matter through the interphase surface, but also determine the required length of the device and investigate theoretically the effect of the multicomponent mass transport parameters on the solution of the problem.

NOTATION

C, vector of mole concentrations of the mixture components; C', vector of the concentrations transformed by means of expressions (11); $[D]$, matrix of the multicomponent diffusion coefficients; $\ulcorner D'$, diagonal matrix consisting of eigenvalues of matrix $[D]$; $[G]$ and $[L]$, fundamental matrices of the $[D_g]$ and $[D_\ell]$ matrices; h_0, thickness of the liquid film; $\ulcorner I_\lrcorner$, unit matrix; J_k, molar flux of the k-th component; $[M]$, matrix of phase equilibrium constants; n, number of mixture components; R, half-width of the channel; u and $\bar{u}$, phase velocity and its integral-mean value, respectively; v, dimensionless phase velocity; x and y, coordinates; ξ and η, corresponding dimensionless coordinates; ν, kinematic viscosity coefficient; μ, dynamic viscosity coefficient; 0, null vector; e, unit vector; δ_{ij}, Kronecker symbol; $Re_\ell = 4\bar{u}_\ell/h_0/\nu_\ell$ and $Re_g = 4\bar{u}_g(R-h_0)/\nu_g$, Reynolds numbers for the liquid and the gaseous phase, respectively; $Sc'_k = \nu/D'_k$, modified Schmidt number for the k-th component. Subscripts: 0, value at the inlet of the contact device; e, equilibrium value; k, value for the k-th component; ℓ and g, liquid and gaseous phase, respectively.

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